Feasibility study for tunnel-building interaction by using the analytic solution for a circular tunnel in an elastic-plastic half space

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ABSTRACT: The presentation of the closed-form solution for the problem of the plastic zone and stress distribution around a circular tunnel in an elastic-plastic half space, derived using bipolar coordinates, as well as its implementation in practical examples such as the feasibility study of a tunnel-building interaction, is the main scope of this paper. By assuming a uniformly applied surface loading, the whole semi-infinite space is under uniform pressure, while the plastic zone formation around the circular tunnel is controlled by the applied internal support pressure. The plastic behavior of the half space is described by the Mohr-Coulomb yield criterion and the soil is assumed to be homogeneous and isotropic with earth pressure coefficient K_o equal to unity. In order to study, in a preliminary phase, the tunnel-building interaction problem, this innovative analytic solution is used.

1. INTRODUCTION

Only a few publications describe analytic solutions for tunnelling problems having elastic behaviour and they all concern the stress disturbance of either an equilibrated semi-infinite or an infinite elastic solid due to the presence of a circular cavity (Mindlin, 1939; Muir Wood, 1975; Pender, 1980; Verruijt & Booker, 1996; Verruijt, 1998; Strack, 2002). For the case of elastic-plastic behaviour, the problem of the plastic zone distribution around a circular tunnel has been analyzed by many authors, but only for infinite space (Bray, 1967a, 1967b; Kachanov, 1971; Detournay & John, 1988) which is the case of a very deep tunnel. However the stress disturbance of an elastic-plastic semi-infinite space, pierced by a circular tunnel and bounded by a loaded upper surface, has not yet been analyzed.

A closed from solution of the problem is presented for cohesive-frictional homogeneous isotropic soil with K_o=1.0 (Massinas & Sakellariou, 2009). By using the derived closed form solution the building-tunnel interaction problem has been analyzed in a preliminary phase (feasibility study).

2. ANALYTIC SOLUTION

In the present study, the tunnel is considered to be a horizontal cylindrical cavity of radius r_i with its axis parallel to the z-axis of a rectangular coordinate system x, y, z. The surrounding soil has only an upper boundary, x-axis (the plane y=0), which represents the surface. Such a solid is characterized as a semi-infinite space. The positive direction of y-axis (vertically downward) bisects the circular tunnel and the half-plane (see Fig. 1). A uniform pressure Po acts in the upper boundary (y=0). It is assumed that the earth pressure coefficient K_o is equal to unity, thus vertical stresses are equal to the horizontal stresses. Moreover, the whole half-space is under uniform pressure, thus no gravitational forces are acting (case of zero body force). Finally, it is mentioned that a tension positive notation is used throughout this paper.

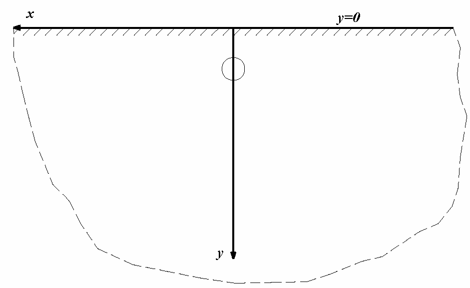

Figure 1. Geometry of the problem

Initially before the tunnel excavation, where an internal uniform pressure P_o acts on the tunnel's periphery, the whole half-space is under uniform stress state $P_{o.}$ When the excavation of the tunnel begins, the internal pressure decreases from its in situ value P_o ($P_i<P_o$), causing elastic stress distribution around the circular cavity. As the internal pressure decreases further to a critical value P_{cr} (a value connected with soil's cohesion and friction) initial yielding occurs at the tunnel wall. After initial yielding at the cavity wall, a plastic zone of unknown shape (to be calculated in the present study) forms around the tunnel and an elastic-plastic interface is created, with further decrease of the internal pressure.

2.1. Bipolar curvilinear coordinate system

Considering the boundaries of the problem (straight and circular), the mathematical analysis will be simplified by using a proper curvilinear coordinate system (α,β) such as the bipolar coordinate system, which is obtained through the conformal transformation of the type

$$x + iy = i\kappa \coth\frac{\alpha+i\beta}{2}, \tag{1}$$

in which $i=\sqrt{-1}$, κ=distance from the origin to a pole.

This kind of system was first applied in two dimensional elasticity by G. B. Jeffery (1921), who gave the general solution of the elasticity equations in plane strain and plane stress conditions, in bipolar coordinates for the case of zero body force. Solving Equation (1) for x, y the following stands

$$x = \frac{\kappa \sin\beta}{\cosh\alpha - \cos\beta}, \tag{2a}$$

$$y = \frac{\kappa \sinh\alpha}{\cosh\alpha - \cos\beta}, \tag{2b}$$

The general scheme of bipolar coordinates is shown in Fig. 2. If the two poles of the system is O_1 and O_2 at the points (0,κ) and (0,-κ) respectively and M any point in the plane and if distances (O_1M), (O_2M) are of lengths l_1, l_2 and are inclined at angles θ_2, θ_1 to x-axis, then $\alpha=\log(l_2/l_1)$ and $\beta=\theta_1-\theta_2$.

The curves α=const are a set of coaxial circles having the poles O_1, O_2 for limiting points. The circles corresponding to positive values of α lie below x-axis (along with the direction of positive y-axis) while those corresponding to negative values above. The centers of these circles lie on the y-axis at distances $d=\kappa\coth\alpha$ and they have radii $r=\kappa\,\mathrm{csch}\,\alpha$. The radical circle $\alpha=0$ gives the x-axis. This and a circle $\alpha=\alpha_i=const$ (positive) represent the boundaries of the problem.

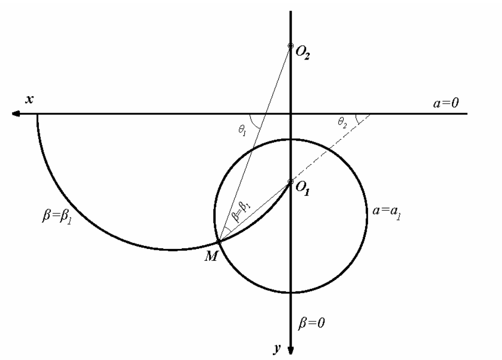

Figure 2. Principle of bipolar coordinate system

The curves β=*const* are circular arcs passing through the poles and cutting the first set of circles orthogonally. β is the angle included between the radius l_1 and l_2. On the right-hand side of the y-axis β is negative and on the left-hand side positive, while on the y-axis β=0, except on the segment (O_1O_2) where $\beta=\pm\pi$. At infinity α=0, β=0 and at the poles O_2, O_1, $\alpha=-\infty$ and $+\infty$ respectively.

The components of stress in bipolar coordinate system were given by Jeffery (1921) as

$$\kappa\sigma_\alpha = \left\{ \begin{array}{l} (\cosh\alpha - \cos\beta)\dfrac{\partial^2}{\partial\beta^2} - \\ -\sinh\alpha\dfrac{\partial}{\partial\alpha} - \\ -\sin\beta\dfrac{\partial}{\partial\beta} + \cosh\alpha \end{array} \right\} \left(\frac{\chi}{J}\right) \tag{3a}$$

$$\kappa\sigma_\beta = \left\{ \begin{array}{l} (\cosh\alpha - \cos\beta)\dfrac{\partial^2}{\partial\alpha^2} - \\ -\sinh\alpha\dfrac{\partial}{\partial\alpha} - \\ -\sin\beta\dfrac{\partial}{\partial\beta} + \cos\beta \end{array} \right\} \left(\frac{\chi}{J}\right) \tag{3b}$$

$$\kappa\tau_{\alpha\beta} = -(\cosh\alpha - \cos\beta)\frac{\partial^2}{\partial\alpha\partial\beta}\left(\frac{\chi}{J}\right) \tag{3c}$$

where $J = \dfrac{\kappa}{\cosh\alpha - \cos\beta}$

2.2. Solution procedure

Let assume that the internal pressure in tunnel's periphery decreases from its in situ value, to P_i ($P_{cr}<P_i<P_o$), causing in such way elastic stress distribution around the circular cavity. Considering the above statement, the following expression for the stress function (as proved by Jeffery, 1921) is adopted

$$\frac{\chi}{J} = B_o\alpha(\cosh\alpha - \cos\beta) + \\ +(A_1\cosh 2\alpha + B_1 + C_1\sinh 2\alpha)\cos\beta \quad (4)$$

Calculating σ_α and σ_β according to equations (3) and applying the boundary conditions, σ_α=-P_i on α=α_i, σ_α=-P_o on α=0 and $\tau_{\alpha\beta}$=0 on both boundaries, the constants (B_o, A_1, C_1, B_1) of the stress function are calculated. By differentiating equation (4) according to equations (3) and after algebraic manipulations, the elastic stresses are determined by Massinas & Sakellariou (2009).

As the internal pressure decreases further to a critical value P_{cr}, initial yielding begins to occur on the tunnel periphery (α=α_i). At this point, the Mohr-Coulomb yield criterion is satisfied

$$\lambda\sigma_\alpha - \sigma_\beta = Y \quad (5)$$

where $\lambda=(1+\sin\varphi)/(1-\sin\varphi)$, $Y=2c\cos\varphi/(1-\sin\varphi)$ and c,φ are soil's cohesion and frictional angle respectively. Using the elastic stresses and the yield criterion, the critical internal pressure is calculated (Massinas & Sakellariou, 2009)

$$P_{cr} = \frac{2\kappa^2}{2(\kappa^2 + r_i^2\sin^2\beta) + \kappa^2(\lambda-1)} \cdot \left\{ P_o\frac{\left(\kappa^2 + r_i^2\sin^2\beta\right)}{\kappa^2} - \frac{Y}{2} \right\} \quad (6)$$

where r_i is the tunnel.

After the initial yielding at the cavity wall, a plastic zone forms around the tunnel (plastic region) and an elastic-plastic interface is created with further decrease of the internal pressure $P_i<P_{cr}$. It is obvious that the elastic-plastic interface α_c will be a function of variable β, $\alpha_c=f(\beta)$, associated with the geometric properties of the problem (tunnel radius and depth from surface), the external uniform pressure P_0 as well as the internal tunnel pressure and finally, the soil's cohesion and friction angle. We assume that inside the plastic zone the trajectories of principal stresses coincide with the bipolar coordinates (Grigoriev, 1968) and as a result the shear stress $\tau_{\alpha\beta}$ is taken equal to zero. Thus, the equilibrium equation inside the zone is written in the form

$$(\cosh a - \cos\beta)\frac{\partial\sigma_a}{\partial\alpha} - (\sigma_a - \sigma_\beta)\sinh a = 0 \quad (7)$$

where σ_α and σ_β are the principal normal stresses. By combining the differential equation with the yield criterion and after solving, we obtain the principal normal stresses in the plastic zone

$$\sigma_{\alpha pl} = \frac{Y}{\lambda-1} + A\left(\cosh\alpha - \cos\beta\right)^{-(\lambda-1)} \quad (8a)$$

$$\sigma_{\beta pl} = \frac{Y}{\lambda-1} + \lambda A\left(\cosh\alpha - \cos\beta\right)^{-(\lambda-1)} \quad (8b)$$

For the determination of constant A the continuity of stress components at the elastic/plastic interface is used

$$\sigma_{\alpha el,c} = \sigma_{\alpha pl,c} = -P_c \quad (9)$$

where P_c is the critical value that limits further extension of the plastic zone and is of the same form as Equation (6), but with r_i replaced by r_c which is the plastic zone radius.

By combining Equations (8a) and (9), the constant A is determined and the final expressions for plastic stresses are derived (Massinas & Sakellariou, 2009).

Finally, by applying $\sigma_{\alpha pl}$,

$$\sigma_{\alpha pl} = \frac{Y}{\lambda-1} - \left(P_c + \frac{Y}{\lambda-1}\right)\left(\frac{r}{r_c}\cdot\frac{d_c - r_c\cos\beta}{d - r\cos\beta}\right)^{\lambda-1} \quad (10)$$

at the cavity wall, the relationship between the internal pressure in the tunnel's periphery and the plastic zone radius r_c is derived (Massinas & Sakellariou, 2009)

$$\left(\frac{r_c}{r_i}\cdot\frac{d_i - r_i\cos\beta}{d_c - r_c\cos\beta}\right)^{1-\lambda} = \frac{\left[2M_o + \kappa^2(\lambda-1)\right]\cdot\left[Y + P_i(\lambda-1)\right]}{2M_o\left[Y + P_o(\lambda-1)\right]} \quad (11)$$

where $M_o = \kappa^2 + r^2_c sin^2\beta$. By using Equation (11) the plastic zone shape can be calculated.

2.3. Solution validation

By using the FLAC program, the results from the computational plane-strain analyses are compared with the results extracted from the closed-from solution. The parameters used in both methods of calculation (FDM and analytic solution), are presented in Table 1. It is evident from the results that the analytic calculation of the plastic zone fits very well with the results from the FDM computational analysis (see Fig. 3).

Table 1. FDM & analytical solution (Massinas et al, 2009)

Analysis	Parameters				
	r_i, m	d_i, m	P_o, kPa	P_i, kPa	c, kPa φ, ^{o}
SMC1	5	10	250	50	60 25
SMC2	5	10	250	30	35 21

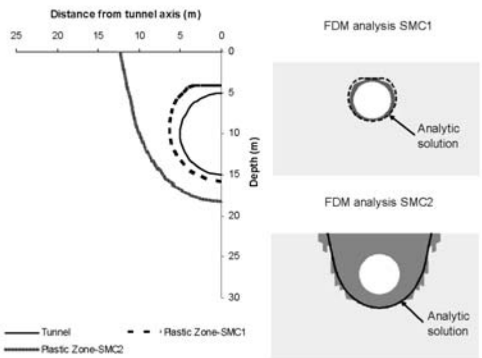

Figure 3. Analytic solution and FDM analysis results: closed-form solution validation (Massinas et al, 2009)

3. FEASIBILTY STUDY FOR TUNNEL-BUILDING INTERACTION

In the present paragraph the problem of the tunnel-building interaction has been examined in a preliminary phase (feasibility study), by using the already presented closed-form solution.

3.1. Introduction of the problem-Geotechnical properties

Assume that a new METRO line, consisted of a 10 m in diameter circular tunnel, is planned to be constructed under a 70-story building. The building reaches a height of 210 m and is founded on a combined pile-raft foundation, with piles of 10 m in length and a raft with a thickness of 4 m. For this kind of building the estimated average pressure (due to building's weight) at the foundation level is B_w=1050 kPa (Sorochan E.A. and Konyukhov D.S, 2005).

The excavation and temporary support of the tunnel is planned to be performed using an earth pressure balance-tunnel boring machine (EPB-TBM): thus temporary support of the tunnel will be provided directly by the EPB-TBM, which is capable of producing pressure with the excavated material and a certain type of foam if needed, in front of the cutter head and around the cavity wall.

The tunnel construction will take place on a bed composed of intercalation of sands of medium fineness, silty sand, grey mucky silty clay and clayey soil layer, with the following general characteristics: γ=20 kN/m^3, cohesion c that varies from 0 to 100 kPa (depend on the soil layer) and friction angle φ=35^o.

The low geotechnical parameters of the soil layers in accordance with the increased weight of the high-rise building determine the "boundaries" of the problem. In order to evaluate the feasibility of a tunnel construction, the proper location (in depth) of the METRO line has to be established and the adequate support pressure has to be calculated. Thus, through parametric analyses the distance of the tunnel from the building's foundation and the appropriate support pressure are calculated, in order to keep the shape of the plastic zone to minimum extension; thus both to reduce the settlement development at the building's foundation level and to ensure the stability of the underground excavation.

3.2. Physical & equivalent model for analytical calculations

For the present feasibility study certain assumptions were taken into consideration. The building is taken into consideration by assuming as an upper boundary of the semi-infinite space its foundation level and applying a uniform surface pressure, infinite in length, equal to building's foundation loading: thus the building's stiffness is neglected. Furthermore, the earth pressure (due to gravity) at the depth of the tunnel is taken into account as additional load, applied uniformly at the upper boundary of the half-space (see Fig. 4).

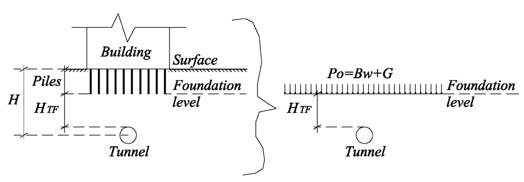

Figure 4. Physical and equivalent model for analytical calculations

Using the closed-form solution of the present study, a quick estimation of the plastic zone shape around the circular tunnel for different tunnel's depths and values of P_i gives a quick estimate of the required EPB-TBM support pressure.

3.3. Analytical calculation results

By using equation (11) different diagrams are constructed showing the plastic zone width at four certain points around the circular tunnel for different values of support pressure: at the crown, 45° from crown, at the sidewalls and below the invert.

For the lower value of the soil's cohesion (c=0 kPa) the plastic zone width is calculated and is presented in the diagram of figure 5, for different values of support pressure. The depth of the centre of the tunnel from the building's foundation level is taken 15 m. Furthermore, the depth of the tunnel's centre from the surface is 25 m: thus for γ=20 kN/m^3 a mean uniform pressure G of 500 kPa is added at the foundation level.

Therefore the total pressure that is uniformly applied at the upper boundary of the half-space (building's foundation level) is consisted of, the building's weight and the earth pressure due to gravity and is given below

$$P_o = B_w + G = 1050 \text{ kPa} + 500 \text{ kPa} = 1550 \text{ kPa} \quad (12)$$

It is evident from the diagram of Fig. 5 that for the lower value of the soil's cohesion and for a support pressure equal to 1 bar, the plastic zone width at the crown is 10 m, thus it propagates up to the foundation level. At the 45° from the crown the plastic zone has a maximum width that exceeds 12 m. At the tunnel's sidewall and invert the plastic zone width is calculated 7 m and 4.5 m respectively.

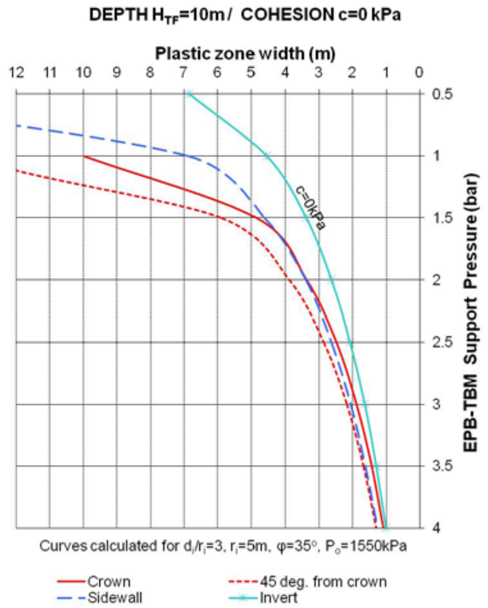

Figure 5. Plastic zone width for P_o=1550 kPa, c=0 kPa & H_{TF}=10 m

It is obvious from the above that in order to keep the shape of the plastic zone to minimum extension (<1 m) a support pressure with mean value greater than 4.5 bar is required from the EPB-TBM.

Moreover, by taking into account the maximum value of soil cohesion (c=100 kPa), the diagram of Fig. 6 is constructed.

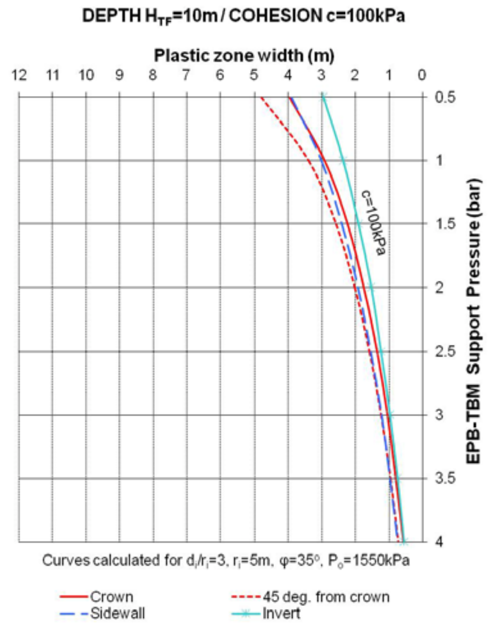

Figure 6. Plastic zone width for P_o=1550 kPa, c=100 kPa & H_{TF}=10 m

In this case for a support pressure of 3 bar the maximum width of the shaped plastic zone does not exceed 1.25 m whereas for support pressure of 3.5 bar the maximum plastic zone width is lower than 1 m. In the diagram of Fig. 7 the above cases (c=0 & 100 kPa) are presented together. Please note that for support pressure of 3 bar in the first case (c=0 kPa) the maximum plastic zone width exceed 2 m.

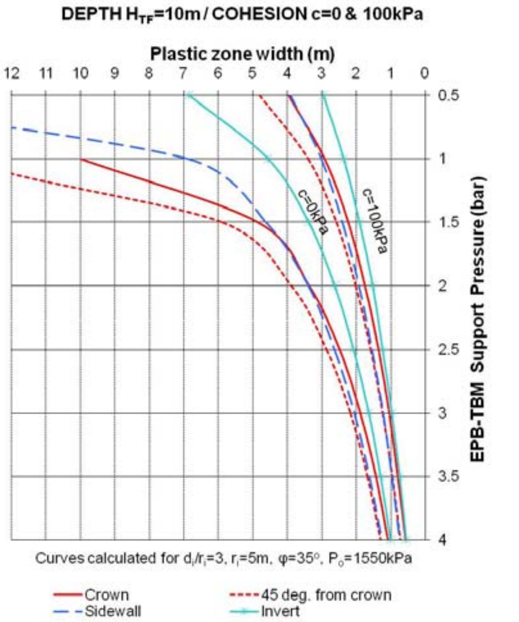

Figure 7. Plastic zone width for P_o=1550 kPa, c=0 & 100 kPa & H_{TF}=10 m

From the diagrams of Fig. 7 is obvious that for small values of support pressure (e.g. 1 bar) the plastic zone shape deviates from absolute circular geometry, whereas for higher values of support pressure (e.g. >3 bar) the plastic zone shape tends to be circular. For the case of c=0 kPa the plastic zone propagates up to the foundation level (for pressure 1 bar).

In order to conclude, for the proper depth of the tunnel and the necessary support pressure to be provided from EPB-TBM a new calculation for the plastic zone is realized by considering the depth of the centre of the tunnel from the building's foundation level at 25 m (35 m from the surface). Considering the new depth of the tunnel, equation (12) takes the following form

$$P_o=B_w+G=1050\text{ kPa} + 700\text{ kPa}=1750\text{ kPa} \quad (13)$$

The calculation was made for soil's cohesion equal to 100 kPa (see Fig. 8).

Due to the deeper tunnel alignment the plastic zone shape has a circular geometry (Massinas et al, 2009). For a support pressure of 3 bar & 3.5 bar, provided by the EPB, the maximum width of the plastic zone is calculated 1.25 m and 1 m respectively.

In Table 2 the general results from the above presented calculations are presented.

Table 2. General results of plastic zone calculations

Parameters-Results					
Support pressure (bar)	Soil's cohesion c (kPa)	Tunnel's centre depth from foundation level (m)	Tunnel's centre depth from surface (m)	Plastic zone shape	Max. plastic zone width (m)
>4.5	0	15	25	almost circle	<1.0
3.0	100	15	25	almost circle	1.25
3.5	100	15	25	almost circle	1.0
3.0	100	25	35	circle	1.25
3.5	100	25	35	circle	1.0

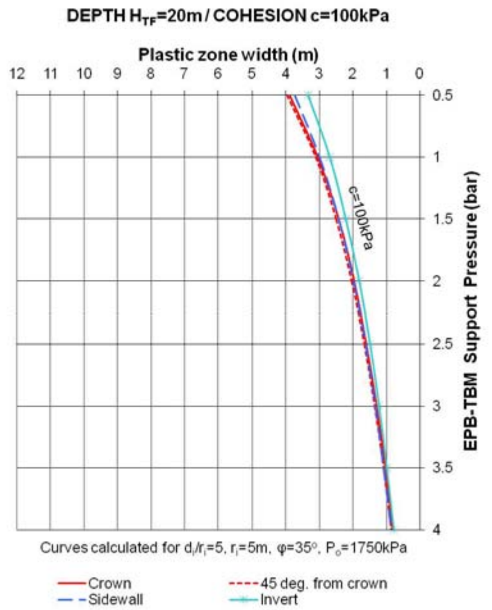

Figure 8. Plastic zone width for P_o=1750 kPa, c=100 kPa & H_{TF}=20 m

3.4. Conclusions

The excavation and temporary support of a circular tunnel under an existing high-rise building and inside a soil with low geotechnical parameters is a problem that is examined in the present paper.

According to the calculation results that are presented synoptically in Table 2 the main parameter that affects the construction of the tunnel are the cohesion of the soil and the depth of the tunnel alignment below the building's foundation level.

These two parameters determine the support pressure that has to be provided from the EPB boring machine in order to keep the shape of the plastic zone to minimum extension; thus both to reduce the settlement development at the building's foundation level and to ensure the stability of the underground excavation.

In the light of the above and taking into consideration that the soil's cohesion varies from 0 to 100 kPa we conclude to the fact that special ground improvement, such as jet grouting, must be provided in the area where the tunnel excavation will take place. With this improvement measure the cohesion of the soil has to be at least 100 kPa at the area of the tunnel excavation, in order for the EPB-TBM to operate with an efficient support pressure (~3.0 bar).

Concerning the longitudinal alignment, the crown of the tunnel is feasible to pass 10 m (H_{TF}) below the foundation level of the building. As it is evident from the calculation results there is no need for a deeper longitudinal alignment. On the contrary, a deeper alignment may cause difficulties in the ground improvement works.

Concluding, for an improved soil with cohesion greater than 100 kPa, only minimum plastic flow will occur around the circular tunnel for a support pressure of ~3.0 bar. Thus minimum settlements will occur at the building's foundation level.

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